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Open string/open D-brane dualities: old and new

Henric Larsson

Institute for Theoretical Physics
Göteborg University, and
Chalmers University of Technology
SE-412 96 Göteborg, Sweden
E-mail: solo@fy.chalmers.se

Per Sundell

Institute for Physics, University of Groningen
Nijenborgh 4, 9747 AG Groningen, The Netherlands
E-mail: p.sundell@phys.rug.nl

Abstract: We examine magnetic and electric near horizon regions of maximally supersymmetric D-brane and NS5-brane bound states and find transformations between near horizon regions with worldvolume dual magnetic and electric fluxes. These point to dual formulations of NCYM, NCOS and ODp theories in the limit of weak coupling and large spatial or temporal non-commutativity length scale in terms of weakly coupled theories with fixed worldvolume dual non-commutativity based on open D-branes. We also examine the strong coupling behavior of the open D-brane theories and propose a unified web of dualities involving strong/weak coupling as well as large/small non-commutativity scale.

Keywords: Non-perturbative Effects, AdS/CFT Correspondance, Non-Commutative Geometry, Supersymmetry and Duality.

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Contents

1. Introduction

In this paper we examine dualities between decoupled non-gravitational theories arising on branes in M-theory and string theory [1]–[17]. These arise in limits involving taking the Planck length to zero at fixed energies, or equivalently, the energy to zero at fixed Planck length, while keeping a finite effective non-gravitational coupling on the brane. Starting from a large stack of branes, whose decoupled dynamics is described by the residual gravitational M-theory/string theory in its near horizon region, the dynamics of a single or a few probe branes is obtained by separating them from the stack and moving them up and away from the gravitational core of the near horizon region into a UV region where the gravitational energy scale, set by a running Planck mass, diverges. This process is of course equivalent to scaling the Planck length to zero in a flat background, but it has the advantage of being embedded in a non-perturbative setup. The decoupling of the probe brane then follows from the decoupling of the near horizon region from the flat region. Moreover, the required scalings of tensions and fluxes are easily read off from the components of the near horizon configuration.

A further advantage is that the underlying U-duality group descends to the decoupled theories. Discrete elements of the group (S-dualities, T-dualities and uplifts) generate dualities between decoupled theories which differ in their perturbative spectrum. Continuous elements generate "non-commutative deformations". A magnetic deformation affects the geometry of the worldvolume, but does not alter the perturbative spectrum and couplings. Electric deformations act differently. A finite electric field becomes critical in the decoupling limit which leads to a decoupled theory described in terms of an open brane. Thus, loosely speaking, there are two kinds of decoupled non-gravitational theories: "field theories" that arise in non-critical limits (and their magnetic deformations) and "open brane theories" that arise in critical limits (and their magnetic deformations).

M-theory has a critical open-membrane (OM) theory on its five-brane [1]–[5], which subsumes various spatially non-commutative Yang-Mills (NCYM) theories [6]–[11] and critical non-commutative open string (NCOS) theories [12, 13, 14] on the D-branes. While OM theory is non perturbative, NCYM and NCOS theories have dimensionless couplings which are determined from the geometrical data of the wrapping of the OM theory expressed using the six-dimensional open membrane metric [2]. The OM theory also accounts for a non-commutative extension of the type (2,0) little string theory (LST) on the IIA NS5-brane containing heavy D2-branes called OD2 theory [1, 4]. T-duality in a direction parallel to the electric RR flux sends the OD2 theory to a non-commutative extension of the type (1,1) LST on the IIB NS5-brane referred to as the OD1 theory [1, 4], that in turn is S-dual to the six-dimensional NCOS theory on the IIB D5-brane. The OD2 theory can also be T-dualized along a magnetic circle to yield OD3 theory on the IIB NS5-brane [1, 15, 16]. In what follows we shall assume that all brane directions are non compact — compact directions lead to decoupled theories involving open brane modes as well as wound closed strings and wrapped higher-dimensional branes with strictly positive winding/wrapping number [17].

The above theories are thus related by dualities involving various limits in the coupling and the non-commutative length scale. There are, however, several limits in which none of the above theories provides a perturbative definition. For example, four-dimensional NCYM theory with large $\theta_{\rm YM}$ and small $g^2_{\rm YM}$ and fixed NCYM soliton tension $(g^2_{\rm YM}\theta_{\rm YM})^{-1}$, or equivalently four-dimensional NCOS with fixed $\alpha'_{\rm eff}$ and large $G^2_{\rm OS}$, should be dual to a theory with a well-defined expansion in $g^2_{\rm YM}=1/G^2_{\rm OS}$ at fixed $\alpha'_{\rm eff}=\theta_{\rm YM}g^2_{\rm YM}$. There are at least four possibilities for what interactions this theory may have. They may be the ones of the large $\theta_{\rm YM}$ and small $g^2_{\rm YM}$ limit of NCYM theory. It may also have weak self-interactions of NCOS type provided this theory is actually self-dual, though this seems artificial from the underlying IIB theory perspective as its self-duality simply maps the NCYM supergravity dual to a strongly coupled NCOS dual. The theory may also be free.

A fourth option is that the theory has non-trivial interactions different from both NCOS and NCYM theory. The supergravity dual suggests interactions based on critical open D-strings. From the NCYM viewpoint a finite $\theta_{\rm YM}$ tilts an open D-string a finite angle [18, 19]. Such a sub-critical open D-string casts a "shadow" on the D3-brane giving rise to a non-local NCYM soliton. The F-strings of the NCOS theory become critical in a discontinuous way whereby the decoupled open F-string "dipoles" arise only in the critical limit whereas any sub-critical electric field leads to essentially the same coupled open-closed string dynamics. Similarly, the critical transition of the open D-string arises in the limit of infinite $\theta_{\rm YM}$ when it coincides with its shadow. Then new massless states associated with open F-strings of zero length stretched between the open D-string and the D3-brane appears in the spectrum. Therefore the usage of the term "critical open D-brane" requires extra care when meant to suggest an analogy with critical open F-strings. In this paper we shall not examine the microscopic aspects of this issue any further. We shall adopt the point of view, however, that there exists some non-trivial theory, which we shall denote by $\widetilde{\rm OD1}$, that has a well-defined perturbative expansion in $G^2_{\widetilde{\rm OD1}}=g^2_{\rm YM}$ at fixed $\ell^2_{\widetilde{\rm OD1}}=\theta_{\rm YM}g^2_{\rm YM}$ (thus involving critical open D-strings) and that has a supergravity dual obtained by taking the electric near horizon limit of a D3-brane with electrically deformed RR two-form potential (see below for details). We also wish to remark[1] that from the macroscopic point of view the usual two-dimensional phase diagram in the energy-coupling plane does not contain the new $\widetilde{\rm OD1}$ phase, as this diagram contains the phases of the theory expanded around its usual vacuum with zero number of non-commutative solitons. It is expected, however, that the

[1]We are thankful to A. Hashimoto for pointing this out to us.

$\widetilde{\text{OD1}}$ theory arises as a stable thermodynamic phase provided a chemical potential for the solitons is added, such that the theory can be examined in an environment with finite expectation value for the soliton number.

Having made this assumption, it is natural to extend the notion of open Dq-brane theories in $(q+3)$ dimensions to $q = 0, 1, 2, 3$. We shall thus discuss $\widetilde{\text{OD}q}$ theories based on D$(q+2)$-brane supergravity duals with electric RR $(q+1)$-form flux and that we assume have a well-defined perturbative expansion in a coupling, to be specified below, at fixed temporal non-commutativity equal to a critical open Dq-brane tension (see above remark). We shall also discuss generalized gauge theories, which we shall refer to as Dq-GT, based on D$(q+2)$-brane supergravity duals with magnetic RR $(q+1)$-form flux and that we assume have a well-defined perturbative expansion at fixed spatial non-commutativity. The Dq-GT also have analogs living on NS5-branes, which we shall refer to as Dq-$\widetilde{\text{GT}}$, and that are generalized gauge theories in the IIB theory and tensor counterparts in IIA. In fact, the Dq-$\widetilde{\text{GT}}$ are magnetic deformations of the tensionless little string theories on the type-IIA/B NS5-branes [20], which may serve as a separate motivation for including these theories. Though we shall base all our observations on the Dq-GT and Dq-$\widetilde{\text{GT}}$ on supergravity duals, we wish to think of them as zero-slope limits of open D-brane theories similar to those yielding NCYM theories from open strings.

In the remainder of this paper we shall examine the supergravity duals of these theories and deduce an extended and unified duality web which incorporates limits of large couplings as well as large spatial and temporal non-commutativity length scales.

The plan of the paper is as follows: in section 2 we give the supergravity solutions with special attention paid to the different parameterizations resulting from electric and magnetic deformations. In section 3 we show the equivalence between electric and dual magnetic near horizon regions. The new open D-brane theories are examined in sections 4 and 5. Section 6 contains a summary and discussion. The conventions used for S-duality, T-duality and uplifting to 11-dimensions are given in appendix A.

2. The supergravity solutions

In this section we obtain each of the half-supersymmetric Dp-F1, D$(q+2)$-Dq and NS5-Dp bound states by either a magnetic or a worldvolume-dual electric deformation. We then obtain the supergravity duals by taking magnetic or electric near horizon limits, respectively, after which we define couplings and length scales. These identifications shall be discussed in more detail in sections 4 and 5.

2.1 NS-deformations of Dp-branes (NCOS and NCYM)

We start by obtaining the Dp-F1 and Dp-D$(p-2)$ bound states by deforming a half-supersymmetric Dp-brane with an electric B-field and a magnetic B-field, respectively. The resulting electric bound states are given by[2]

$$
\begin{aligned}
ds^2 &= H^{-1/2}\left(\frac{1}{h_-}\left(-dx_0^2+dx_1^2\right)+dx_2^2+\cdots+dx_p^2\right)+\\
&\quad + H^{1/2}\left(dr^2+r^2 d\Omega_{8-p}^2\right),\\
e^{2\phi} &= \frac{1}{h_-} g^2 H^{\frac{3-p}{2}},\\
B &= \frac{\theta}{\alpha' H h_-} dx^0\wedge dx^1,\\
C &= \frac{1}{gHh_-} dx^0\wedge\cdots\wedge dx^p + (7-p)N(\alpha')^{\frac{7-p}{2}}(1-B)\wedge\epsilon_{7-p} -\\
&\quad - \frac{\theta}{gH\alpha'} dx^2\wedge\cdots\wedge dx^p,\\
\text{where}\qquad H &= 1+\frac{gN(\alpha')^{\frac{7-p}{2}}}{r^{7-p}},
\end{aligned}
\tag{2.1}
$$

and the magnetic ones by

$$
\begin{aligned}
ds^2 &= H^{-1/2}\left(-dx_0^2+\cdots+dx_{p-2}^2+\frac{1}{h_+}\left(dx_{p-1}^2+dx_p^2\right)\right)+\\
&\quad + H^{1/2}\left(dr^2+r^2 d\Omega_{8-p}^2\right),\\
e^{2\phi} &= \frac{1}{h_+} g^2 H^{\frac{3-p}{2}},\\
B &= -\frac{\theta}{\alpha' H h_+} dx^{p-1}\wedge dx^p,\\
C &= \frac{1}{gHh_+} dx^0\wedge\cdots\wedge dx^p + (7-p)N(\alpha')^{\frac{7-p}{2}}(1-B)\wedge\epsilon_{7-p} -\\
&\quad - \frac{\theta}{gH\alpha'} dx^0\wedge dx^1\wedge\cdots\wedge dx^{p-2},\\
\text{where}\qquad H &= 1+\frac{gN(\alpha')^{\frac{7-p}{2}}}{r^{7-p}}.
\end{aligned}
\tag{2.2}
$$

Here $d\epsilon_{7-p}$ is the volume element on the transverse $(8-p)$-sphere and we have defined:

$$
h_\pm \equiv 1 \pm \left(\frac{\theta}{\alpha'}\right)^2 H^{-1}.
\tag{2.3}
$$

The supergravity duals of the NCOS and NCYM theories are near-horizon regions of these solutions where ds^2/α', B/α' and $C_{(k)}/(\alpha')^{\frac{k+1}{2}}$ $(k=0,1,2,\ldots)$ are held fixed

[2]There are many ways of constructing these solutions. For a general method based on a parametrization directly in terms of the $O(p+1,p+1)$ generators we refer to [21, 22]. For other methods of constructing bound states, see e.g. [23]–[28].

in the limit $\alpha' \to 0$. This can be achieved by keeping the following quantities fixed:[3]

$$\text{Electric near-horizon :} \quad \tilde{x}^\mu = \frac{\tilde{\ell} x^\mu}{\sqrt{\alpha'}}\,, \qquad \tilde{r} = \frac{\tilde{\ell} r}{\sqrt{\alpha'}}\,, \qquad \frac{\theta}{\alpha'} = 1,\, g,\, \tilde{\ell}\,. \tag{2.4}$$

$$\text{Magnetic near-horizon :} \quad x^\mu,\, u = \frac{r}{\alpha'},\, \theta, \begin{cases} \ell^{p-3} = g(\alpha')^{\frac{p-3}{2}} & p \neq 3\,, \\ g & p = 3\,. \end{cases} \tag{2.5}$$

Here ℓ and $\tilde{\ell}$ are fixed length scales. The electric near horizon region of (2.1) gives the supergravity dual of the $(p+1)$-dimensional NCOS theory:

$$\begin{aligned}
\frac{ds^2}{\alpha'} &= \frac{1}{\tilde{\ell}^2} H^{-1/2} \left[H \left(\frac{\tilde{r}}{\tilde{R}}\right)^{7-p} \left(-d\tilde{x}_0^2 + d\tilde{x}_1^2\right) + d\tilde{x}_2^2 + \cdots + d\tilde{x}_p^2 + \right. \\
&\qquad \left. + H(d\tilde{r}^2 + \tilde{r}^2 d\Omega_{8-p}^2) \right], \\
e^{2\phi} &= g^2 H^{\frac{5-p}{2}} \left(\frac{\tilde{r}}{\tilde{R}}\right)^{7-p}, \\
\frac{C_{01\cdots p}}{(\alpha')^{\frac{p+1}{2}}} &= \frac{1}{g\tilde{\ell}^{p+1}} \left(\frac{\tilde{r}}{\tilde{R}}\right)^{7-p}, \\
\frac{C_{23\cdots p}}{(\alpha')^{\frac{p-1}{2}}} &= -\frac{1}{g\tilde{\ell}^{p-1}} \frac{1}{H}, \\
\frac{B_{01}}{\alpha'} &= \frac{1}{\tilde{\ell}^2} \left(\frac{\tilde{r}}{\tilde{R}}\right)^{7-p}, \\
\text{where} \qquad H &= 1 + \left(\frac{\tilde{R}}{\tilde{r}}\right)^{7-p}, \qquad \tilde{R}^{7-p} = Ng\tilde{\ell}^{7-p}\,.
\end{aligned} \tag{2.6}$$

The NCOS length scale and coupling are given by

$$\alpha'_{\text{eff}} = \tilde{\ell}^2\,, \qquad G_{\text{OS}}^2 = g\,. \tag{2.7}$$

The magnetic near horizon region of (2.2) gives the supergravity dual of the $(p+1)$-dimensional NCYM theory ($p \neq 3$; for $p = 3$ we replace ℓ^{p-3} by g):

$$\begin{aligned}
\frac{ds^2}{\alpha'} &= \left(\frac{u}{R}\right)^{\frac{7-p}{2}} \left(-dx_0^2 + \cdots + dx_{p-2}^2 + \frac{1}{h}\left(dx_{p-1}^2 + dx_p^2\right)\right) + \\
&\quad + \left(\frac{R}{u}\right)^{\frac{7-p}{2}} \left(du^2 + u^2 d\Omega_{8-p}^2\right),
\end{aligned}$$

[3]In the electric case (2.4) leads to that α' cancels out, so that the limit $\alpha' \to 0$ becomes trivial. The electric limit is therefore equivalent to setting $\tilde{\ell} = \sqrt{\alpha'}$, and keeping $\tilde{\ell}$, g and all coordinates fixed while taking $\theta \to \alpha'$. In the magnetic case, we keep g fixed in (2.5) when $p = 3$. This means that ℓ^{p-3} or ℓ^{q-1} should be changed to g when $p = q + 2 = 3$ in (2.8) and (2.17).

$$
\begin{aligned}
e^{2\phi} &= \ell^{2p-6} h^{-1} \left(\frac{R}{u}\right)^{(7-p)(3-p)/2}, \\
\frac{C_{01\cdots p}}{(\alpha')^{\frac{p+1}{2}}} &= \frac{1}{\ell^{p-3} h} \left(\frac{u}{R}\right)^{7-p}, \\
\frac{C_{01\cdots p-2}}{(\alpha')^{\frac{p-1}{2}}} &= -\frac{\theta}{\ell^{p-3}} \left(\frac{u}{R}\right)^{7-p}, \\
\frac{B_{p-1,p}}{\alpha'} &= -\frac{\theta}{h} \left(\frac{u}{R}\right)^{7-p}, \\
\text{where} \qquad h &= 1 + \theta^2 \left(\frac{u}{R}\right)^{7-p}, \qquad R^{7-p} = \ell^{p-3} N .
\end{aligned} \tag{2.8}
$$

The Yang-Mills coupling and non-commutativity parameter are identified as

$$
g^2_{\mathrm{YM}} = \begin{cases} \ell^{p-3} & p \neq 3 , \\ g & p = 3 , \end{cases} \qquad \theta_{\mathrm{YM}} = \theta . \tag{2.9}
$$

We remark that the non-renormalizability of NCYM theory for $p > 3$ corresponds to large open string coupling in the UV, signaling large open string fluctuations. For $p = 3$ the open string coupling remains finite and equal to g^2_{YM}, while the tension still diverges so that the massive open string modes decouple. For $p = 2$ the open string coupling goes to zero in the UV. Instead it diverges in the IR, so that the interacting three-dimensional Yang-Mills theory is defined in the IR-limit.

2.2 Electric RR-deformation of NS5-branes (ODp)

The half-supersymmetric NS5-Dp bound states can be obtained by first S-dualizing the D5-F1 solution (2.1) and then applying a series of T-dualities. This is equivalent to starting from a NS5-brane and turning on an electric $(p+1)$-form RR-potential $C_{01\cdots p}$. After rescaling[4] $x^\mu \to \sqrt{g} x^\mu$ and $r \to \sqrt{g} r$ and defining $\tilde{g} = g^{-1}$ we find:

$$
\begin{aligned}
ds^2 &= h_-^{1/2} \left[\frac{1}{h_-} \left(-dx_0^2 + \cdots + dx_p^2 \right) + dx_{p+1}^2 + \cdots + dx_5^2 + \right. \\
&\qquad \left. + H \left(dr^2 + r^2 d\Omega_3^2 \right) \right], \\
e^{2\phi} &= \tilde{g}^2 H h_-^{\frac{3-p}{2}}, \\
C_{01\cdots p} &= -\frac{\theta}{\alpha' \tilde{g} H h_-}, \\
C_{p+1\cdots 5} &= -\frac{\theta}{\alpha' \tilde{g} H}, \\
B &= \alpha' 2N \epsilon_2 , \\
\text{where} \qquad H &= 1 + \frac{N\alpha'}{r^2} .
\end{aligned} \tag{2.10}
$$

[4] The rescaling of the coordinates is part of the solution generation. However, in the identification of two supergravity duals, it is more natural to keep the coordinates fixed and instead rescale the worldvolume parameters; see also comment in section 3.

Taking the electric near horizon limit (2.4) gives the supergravity duals of the ODp theories:

$$
\begin{aligned}
\frac{ds^2}{\alpha'} &= \frac{1}{\tilde{\ell}^2 H^{1/2}} \frac{\tilde{R}}{\tilde{r}} \left[H \left(\frac{\tilde{r}}{\tilde{R}}\right)^2 \left(-d\tilde{x}_0^2 + \cdots + d\tilde{x}_p^2\right) + d\tilde{x}_{p+1}^2 + \cdots + d\tilde{x}_5^2 + \right. \\
&\qquad \left. + H \left(d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2\right) \right], \\
e^{2\phi} &= \tilde{g}^2 H^{\frac{p-1}{2}} \left(\frac{\tilde{r}}{\tilde{R}}\right)^{p-3}, \\
\frac{C_{01\cdots p}}{(\alpha')^{\frac{p+1}{2}}} &= -\frac{1}{\tilde{\ell}^{p+1}\tilde{g}} \left(\frac{\tilde{r}}{\tilde{R}}\right)^2, \\
\frac{C_{p+1\cdots 5}}{(\alpha')^{\frac{5-p}{2}}} &= -\frac{1}{\tilde{\ell}^{5-p}\tilde{g} H}, \\
\frac{B}{\alpha'} &= \frac{2\tilde{R}^2 \epsilon_2}{\tilde{\ell}^2} \\
\text{where} \qquad H &= 1 + \frac{\tilde{R}^2}{\tilde{r}^2}, \qquad \tilde{R}^2 = N\tilde{\ell}^2 .
\end{aligned}
\tag{2.11}
$$

The little string length and ODp length and coupling are defined as [1, 4]:

$$
\ell_{\mathrm{LST}} = \tilde{\ell}, \qquad \ell_{\mathrm{OD}p} = \tilde{g}^{\frac{1}{p+1}} \tilde{\ell}, \qquad G^2_{\mathrm{OD}p} = \tilde{g} . \tag{2.12}
$$

2.3 Electric RR-deformation of D-branes ($\widetilde{\mathrm{OD}q}$)

Half-supersymmetric D$(q+2)$-Dq bound states can be obtained by S-dualizing the D3-F1 bound state (2.1) and then T-dualizing (together with transverse smearings/localizations). This procedure is equivalent to starting with a D$(q+2)$-brane and turning on an electric $(q+1)$-form RR-potential $C_{01\cdots q}$. After rescaling $x^\mu \to \sqrt{g}x^\mu$, $r \to \sqrt{g}r$ and sending $g \to 1/g$ we find:

$$
\begin{aligned}
ds^2 &= h_-^{1/2} \left[H^{-1/2} \left(\frac{1}{h_-} \left(-dx_0^2 + \cdots + dx_q^2\right) + dx_{q+1}^2 + dx_{q+2}^2 \right) + \right. \\
&\qquad \left. + H^{1/2} \left(dr^2 + r^2 d\Omega_{6-q}^2\right) \right], \\
e^{2\phi} &= g^2 H^{\frac{1-q}{2}} h_-^{\frac{3-q}{2}}, \\
C_{01\cdots q+2} &= \frac{1}{gH}, \\
C_{01\cdots q} &= -\frac{\theta}{\alpha' g H h_-}, \\
B_{q+1,q+2} &= -\frac{\theta}{\alpha' H}, \\
\text{where} \qquad H &= 1 + \frac{gN\alpha'^{\frac{5-q}{2}}}{r^{5-q}} .
\end{aligned}
\tag{2.13}
$$

The electrical near horizon limit (2.4) yields the following configuration, which we shall interpret as the supergravity dual of the $\widetilde{\mathrm{OD}q}$ theory:

$$
\begin{aligned}
\frac{ds^2}{\alpha'} &= \frac{1}{\tilde{\ell}^2 H}\left(\frac{\tilde{R}}{\tilde{r}}\right)^{\frac{5-q}{2}}\left[H\left(\frac{\tilde{r}}{\tilde{R}}\right)^{5-q}\left(-d\tilde{x}_0^2+\cdots+d\tilde{x}_q^2\right)+d\tilde{x}_{q+1}^2+d\tilde{x}_{q+2}^2+\right.\\
&\qquad\left.+H\left(d\tilde{r}^2+\tilde{r}^2 d\Omega_{6-q}^2\right)\right],\\
e^{2\phi} &= \frac{g^2}{H}\left(\frac{\tilde{R}}{\tilde{r}}\right)^{(5-q)(3-q)/2},\\
\frac{C_{01\cdots q+2}}{(\alpha')^{\frac{q+3}{2}}} &= \frac{1}{g\tilde{\ell}^{q+3}H},\\
\frac{C_{01\cdots q}}{(\alpha')^{\frac{q+1}{2}}} &= -\frac{1}{g\tilde{\ell}^{q+1}}\left(\frac{\tilde{r}}{\tilde{R}}\right)^{5-q},\\
\frac{B_{q+1,q+2}}{\alpha'} &= -\frac{1}{\tilde{\ell}^2 H},\\
\text{where}\qquad H &= 1+\frac{\tilde{R}^{5-q}}{\tilde{r}^{5-q}},\qquad \tilde{R}^{5-q}=Ng\tilde{\ell}^{5-q}\,.
\end{aligned}
\tag{2.14}
$$

We identify the $\widetilde{\mathrm{OD}q}$ length and coupling as follows

$$
\ell_{\widetilde{\mathrm{OD}q}} = g^{\frac{1}{q+1}}\tilde{\ell}\,,\qquad G^2_{\widetilde{\mathrm{OD}q}} = g\,. \tag{2.15}
$$

2.4 Magnetic RR-deformation of D-branes (Dq-GT)

In this section we deform D(q+2)-branes with magnetic RR (q+1)-form potentials. In section 5.1 we shall argue that the magnetic near horizon region of the resulting half-supersymmetric D($q+2$)-F1 bound state describes a generalized non-commutative gauge theory. The bound states can be obtained by S-dualizing a magnetically (NS) deformed D3-brane and then T-dualizing (together with smearings/localizations). After rescaling $x^\mu \to \sqrt{g}x^\mu$, $r \to \sqrt{g}r$ and sending $g \to 1/g$ we find:

$$
\begin{aligned}
ds^2 &= h_+^{1/2}\left[H^{-1/2}\left(-dx_0^2+dx_1^2+\frac{1}{h_+}\left(dx_2^2+\cdots+dx_{q+2}^2\right)\right)+\right.\\
&\qquad\left.+H^{1/2}\left(dr^2+r^2 d\Omega_{6-q}^2\right)\right],\\
e^{2\phi} &= g^2 H^{\frac{1-q}{2}} h_+^{\frac{3-q}{2}},\\
C_{01\cdots q+2} &= \frac{1}{gH},\\
C_{2\cdots q+2} &= \frac{\theta}{\alpha' g H h_+},
\end{aligned}
$$

$$B_{01} = -\frac{\theta}{\alpha' H}\,,$$

$$\text{where} \qquad H = 1 + \frac{gN\alpha'^{\frac{5-q}{2}}}{r^{5-q}}\,. \tag{2.16}$$

Taking the magnetic near horizon limit (2.5) gives the Dq-GT dual ($q \neq 1$; for $q = 1$ we replace ℓ^{q-1} by g):

$$\begin{aligned}
\frac{ds^2}{\alpha'} &= h^{1/2}\Bigg[\left(\frac{u}{R}\right)^{\frac{5-q}{2}}\left(-dx_0^2 + dx_1^2 + \frac{1}{h}(dx_2^2 + \cdots + dx_{q+2}^2)\right) + \\
&\qquad + \left(\frac{R}{u}\right)^{\frac{5-q}{2}}\left(du^2 + u^2 d\Omega_{6-q}^2\right)\Bigg]\,,\\
e^{2\phi} &= \ell^{2(q-1)} h^{\frac{3-q}{2}}\left(\frac{R}{u}\right)^{(5-q)(1-q)/2}\,,\\
\frac{C_{01\cdots q+2}}{(\alpha')^{\frac{q+3}{2}}} &= \ell^{1-q}\left(\frac{u}{R}\right)^{5-q}\,,\\
\frac{C_{2\cdots q+2}}{(\alpha')^{\frac{q+1}{2}}} &= \frac{\ell^{1-q}\theta}{h}\left(\frac{u}{R}\right)^{5-q}\,,\\
\frac{B_{01}}{\alpha'} &= -\theta\left(\frac{u}{R}\right)^{5-q}\,,\\
\text{where} \qquad h &= 1 + \theta^2\left(\frac{u}{R}\right)^{5-q}\,, \qquad R^{5-q} = N\ell^{q-1}\,.
\end{aligned} \tag{2.17}$$

We identify the Dq-GT coupling and non-commutativity parameter as follows:

$$g^2_{\mathrm{D}q\text{-GT}} = \begin{cases} \ell^{q-1} & q \neq 1\,, \\ g & q = 1\,. \end{cases} \qquad \theta_{\mathrm{D}q\text{-GT}} = \ell^{q-1}\theta\,. \tag{2.18}$$

2.5 Magnetic RR-deformation of NS5-branes (Dq-$\widetilde{\mathrm{GT}}$)

The generalized gauge theories on NS5-branes, which we shall denote by Dq-$\widetilde{\mathrm{GT}}$ (q even for type IIA and q odd for type IIB), arise in the near horizon region of an NS5-brane with a magnetic $(q+1)$-form RR potential that is fixed in the UV. These solutions are constructed by S-dualizing (2.16) for $p = 5$ and then T-dualizing, which gives the following NS5-D$(4-q)$ bound states after the rescalings $x^\mu \to \sqrt{g}x^\mu$ and $r \to \sqrt{g}r$, $\theta \to -\theta$ and setting $g \to 1/g$:

$$\begin{aligned}
ds^2 &= h_+^{1/2}\Bigg(-dx_0^2 + \cdots + dx_{4-q}^2 + \frac{1}{h_+}\left(dx_{5-q}^2 + \cdots + dx_5^2\right) + \\
&\qquad + H(dr^2 + r^2 d\Omega_3^2)\Bigg)\,,\\
e^{2\phi} &= g^2 H h_+^{\frac{3-q}{2}}\,,\\
C_{01\cdots 4-q} &= -\frac{\theta}{\alpha' g H}\,,
\end{aligned}$$

$$
\begin{aligned}
C_{5-q\cdots 5} &= -\frac{\theta}{\alpha' g H h_+}\,,\\
B &= \alpha' 2N\epsilon_2\,,\\
\text{where}\qquad H &= 1+\frac{N\alpha'}{r^2}\,.
\end{aligned}
\tag{2.19}
$$

For $p=5$ the magnetic near horizon limit (2.5) on the D5-brane amounts to keeping $g_{\rm YM}^2=\alpha' g$ fixed, which in turn implies that $g\sim\alpha'^{-1}$. The manipulations used in obtaining (2.19) imply that the magnetic near-horizon limit of (2.19) amounts to taking $\alpha'\to 0$ keeping the following quantities fixed:

$$
\text{Magnetic NS5 near-horizon}:\quad \hat{x}^\mu=\frac{\hat{\ell}x^\mu}{\sqrt{\alpha'}},\qquad \hat{u}=\frac{\hat{\ell}r}{(\alpha')^{3/2}},\ \theta,\ \hat{\ell}^2=g^{-1}\alpha'\,. \tag{2.20}
$$

The resulting magnetic near-horizon region becomes

$$
\begin{aligned}
\frac{ds^2}{\alpha'} &= \frac{h^{1/2}}{\hat{\ell}^2}\Bigg(-d\hat{x}_0^2+\cdots+d\hat{x}_{4-q}^2+\frac{1}{h}\left(d\hat{x}_{5-q}^2+\cdots+d\hat{x}_5^2\right)+\\
&\qquad\qquad +\left(\frac{\hat{R}}{\hat{u}}\right)^2\left(d\hat{u}^2+\hat{u}^2d\Omega_3^2\right)\Bigg)\,,\\
e^{2\phi} &= \frac{h^{\frac{3-q}{2}}}{\hat{\ell}^4}\left(\frac{\hat{R}}{\hat{u}}\right)^2,\\
\frac{C_{01\cdots 4-q}}{(\alpha')^{\frac{5-q}{2}}} &= -\theta\hat{\ell}^{q-3}\left(\frac{\hat{u}}{\hat{R}}\right)^2,\\
\frac{C_{5-q\cdots 5}}{(\alpha')^{\frac{q+1}{2}}} &= -\frac{\theta\hat{\ell}^{1-q}}{h}\left(\frac{\hat{u}}{\hat{R}}\right)^2,\\
\frac{B}{\alpha'} &= \frac{2\hat{R}^2\epsilon_2}{\hat{\ell}^2}\,,\\
\text{where}\qquad h &= 1+\theta^2\left(\frac{\hat{u}}{\hat{R}}\right)^2,\qquad \hat{R}^2=N\hat{\ell}^2\,.
\end{aligned}
\tag{2.21}
$$

We identify the Dq-$\widetilde{\rm GT}$ coupling, non-commutativity parameter and little string length scale as

$$
g^2_{{\rm D}q\text{-}\widetilde{\rm GT}}=\frac{1}{\hat{\ell}^2}\,,\qquad \theta_{{\rm D}q\text{-}\widetilde{\rm GT}}=\theta\hat{\ell}^{q-1}\,,\qquad \ell_{\rm LST}=\hat{\ell}\,. \tag{2.22}
$$

2.5.1 Identification of little string lengths

The following comment on (2.11) and (2.21) is in order: at small $\tilde{r}/\tilde{R}$ or $\hat{u}/\hat{R}$ these solutions approach the undeformed near horizon geometry of a NS5-brane, which is the supergravity dual of six-dimensional LST with length scale given by (2.12) and (2.22).

These identifications can be understood as follows [1, 4]: independently of whether the NS5-brane is deformed or not, the dilaton always grows large close to an NS5-brane, that is for small $\tilde{r}/\tilde{R}$ or $\hat{u}/\hat{R}$. In IIB string theory the resulting S-dual D5-brane dynamics is described by six-dimensional Yang-Mills theory with coupling g^2_{YM}, which defines the IIB LST length scale

$$\mathrm{IIB}: \quad \ell^2_{\mathrm{LST}} \equiv g^2_{\mathrm{YM}} \,. \tag{2.23}$$

From the supergravity dual of the ODp theory given in (2.11) and the one of the Dq-$\widetilde{\mathrm{GT}}$ theory given in (2.21) it follows that g^2_{YM} is given by $\tilde{\ell}^2$ and $\hat{\ell}^2$, respectively, so that (2.23) implies the identifications of the little string length scales made in (2.12) and (2.22) for type IIB.

The IIA little strings have a different origin.[5] At low energies the effective $(2,0)$ tensor theory on the NS5-brane flows to a tensor theory on the M-theory five-brane reduced on a large transverse circle S^1_T with radius R_T. In this picture, the little strings are boundaries of open membranes that are wrapped around S^1_T. The tension of the IIA little strings is therefore the same as that of the IIA closed strings, while their strong coupling origin in M-theory is different. The expression for the tension is given by

$$\mathrm{IIA}: \quad \ell^2_{\mathrm{LST}} \equiv R_T^{-1} \ell^3_{11} \,, \tag{2.24}$$

where ℓ_{11} is the eleven-dimensional length scale that is fixed in the near horizon region of the M5-brane (the dilaton cancels out as usual). It follows that ℓ_{LST} is given by $\tilde{\ell}$ in (2.11) and $\hat{\ell}$ in (2.21), which shows (2.12) and (2.22) also for type IIA.

In the above we have identified the ℓ_{LST} at small separations/low energies. The LST on the NS5-branes decouple, on the other hand, provided that the local closed string coupling becomes small (so that IIB little strings are F1-strings "trapped" in the throat of the NS5-brane while IIA little strings are wrapped open M2-branes with a degenerated cylindrical direction). This is expected to happen in the UV regime, as for the OD1 theory, but as we shall see in section 4, extra care is needed in the case of the OD3-LST since the dilaton approaches a constant value in the UV, leaving the possibility of having the $\widetilde{\mathrm{OD3}}$ theory on the D5-brane.

3. Equivalence between electric and magnetic deformations

In the previous section we have obtained near horizon geometries through various magnetic and electric near horizon limits. However, despite the fact that the electric and magnetic near-horizon limits appear to be quite different, it turns out that the resulting near horizon regions actually are equivalent, provided that one relates the

[5] We are thankful for J.P. van der Schaar for pointing this out to us.

JHEP06(2001)008

fixed parameters. The NCOS dual (2.6) and the Dq-GT dual (2.17) are identified as follows ($p = q + 2 \neq 3$):

$$\begin{tikzcd}
\mathrm{D}(q+2) \text{ - el. F1} \arrow[r, dash, "\text{identify}"] \arrow[d, "\substack{\text{electr.}\\ \text{near-hor.}\\ \text{limit}}"'] & \mathrm{D}(q+2) \text{ - magn. D}q \arrow[d, "\substack{\text{magn.}\\ \text{near-hor.}\\ \text{limit}}"] \\
\text{NCOS} \arrow[r, dash, "\text{identify}"] & \mathrm{D}q\text{-GT}
\end{tikzcd}$$

$$x^\mu = \tilde{x}^\mu\,, \qquad u = \frac{\tilde{r}}{\tilde{\ell}^2}\,, \qquad \ell^{p-3} = g\tilde{\ell}^{p-3}\,, \qquad \theta = -\tilde{\ell}^2\,. \tag{3.1}$$

After a change in the sign in the expression for θ, this transformation also shows the equivalence between the $\widetilde{\mathrm{OD}q}$ dual (2.14) and the NCYM dual (2.8):

$$\begin{tikzcd}
\mathrm{D}(q+2) \text{ - el. D}q \arrow[r, dash, "\text{identify}"] \arrow[d, "\substack{\text{electr.}\\ \text{near-hor.}\\ \text{limit}}"'] & \mathrm{D}(q+2) \text{ - magn. F1} \arrow[d, "\substack{\text{magn.}\\ \text{near-hor.}\\ \text{limit}}"] \\
\widetilde{\mathrm{OD}q} \arrow[r, dash, "\text{identify}"] & \text{NCYM}
\end{tikzcd}$$

For $p = q+2 = 3$ the identification of ℓ and $\tilde{\ell}$ in (3.1) is replaced by the identification of g in (2.6) and (2.17). In order to identify the ODp dual (2.11) and the $\mathrm{D}q\text{-}\widetilde{\mathrm{GT}}$ dual (2.21) we instead do the transformation ($p = 4 - q$):

$$\hat{x}^\mu = \tilde{x}^\mu\,, \qquad \hat{\ell} = \tilde{\ell}\,, \qquad \hat{u} = \frac{\tilde{r}}{\theta}\,, \qquad \theta = \tilde{g}\tilde{\ell}^2\,. \tag{3.2}$$

We remark that the brane-coordinates are identified, so the energies in the related worldvolume theories can be compared directly without any rescaling. This choice also implies that the little string theory length scales in the ODp dual (2.11) and the $\mathrm{D}q\text{-}\widetilde{\mathrm{GT}}$ dual (2.21) are the same.

Provided that the decoupling of both electric and magnetic theories makes sense, the above result shows that they are related in some limit, to which we next turn our attention.

4. Open D-brane theories on D-branes

In this section we begin by arguing that the OD3 theory is S-dual to a theory of decoupled, light D3-branes on the D5-brane, which we shall denote by $\widetilde{\mathrm{OD3}}$, that in turn is dual to six-dimensional NCYM theory at weak coupling and large θ_{YM}. T-duality then points to $\widetilde{\mathrm{OD}q}$ theories with similar properties on the D$(q+2)$-branes for $q = 0, 1, 2, 3$.

4.1 Strongly coupled OD3 theory

The supergravity dual of the six-dimensional OD3 theory is given in (2.11) for $p = 3$. The OD3 theory decouples in the UV limit $\tilde{r}/\tilde{R} \to \infty$ and is a theory of little strings and open D3-branes with fixed tensions:

$$T_{\mathrm{LST}} \equiv \frac{1}{\ell^2_{\mathrm{LST}}}\,, \qquad T_{\mathrm{OD3}} \equiv \frac{1}{\ell^4_{\mathrm{OD3}}} = \frac{1}{G^2_{\mathrm{OD3}}\ell^4_{\mathrm{LST}}}\,, \tag{4.1}$$

where G^2_{OD3} is the OD3 coupling constant which is identified in (2.12). For small coupling the OD3-theory has a perturbative description in terms of light little strings in a special non-commutative geometry giving rise to heavy solitonic open D3-branes. For large G^2_{OD3} the situation becomes reversed, so that the perturbative expansion should be given in terms of light open D3-branes. Indeed, upon S-dualizing the OD3 theory supergravity dual (2.11) we find the $\widetilde{\mathrm{OD3}}$ theory ditto (2.14) with the same little string length (we identify the brane coordinates without any rescaling) provided we make the identifications

$$\ell_{\widetilde{\mathrm{OD3}}} = \ell_{\mathrm{OD3}}\,, \qquad G^2_{\widetilde{\mathrm{OD3}}} = \frac{1}{G^2_{\mathrm{OD3}}}\,. \tag{4.2}$$

In the UV limit, the two configurations given in (2.11) and (2.14) have identical scaling behavior (with S-dual parameters), so that it is natural to assume that both the OD3 theory and the $\widetilde{\mathrm{OD3}}$ theory contain little F1 and D1-strings, respectively, as well as open D3-branes, with the main difference that the light objects at weak coupling are the little strings in OD3 theory and the open D3-branes in $\widetilde{\mathrm{OD3}}$ theory. Therefore we propose that the S-dual of the OD3 theory is the $\widetilde{\mathrm{OD3}}$ theory.

In [1, 15, 29] it was argued that OD3 is S-dual to six-dimensional NCYM theory. This is not in contradiction with the above, but we need to refine the statement as follows: the NCYM and $\widetilde{\mathrm{OD3}}$ theories arise in open F1- and D3-pictures, respectively, that are valid in different limits of the parameter space and provide different perturbative descriptions. From (3.1) it follows that the parameters of the two theories are related as follows:

$$\theta_{\mathrm{YM}} = G^{-1}_{\widetilde{\mathrm{OD3}}}\ell^2_{\widetilde{\mathrm{OD3}}}\,, \qquad g^2_{\mathrm{YM}} = G_{\widetilde{\mathrm{OD3}}}\ell^2_{\widetilde{\mathrm{OD3}}}\,. \tag{4.3}$$

The effective t'Hooft coupling and the effective non-commutativity parameter at energy scale E are then given by

$$g^2_{\rm eff} \equiv N g^2_{\rm YM} E^2 = N G_{\widetilde{\rm OD3}} (E\ell_{\widetilde{\rm OD3}})^2 \,, \qquad \theta_{\rm eff} \equiv \theta_{\rm YM} E^2 = G^{-1}_{\widetilde{\rm OD3}} (E\ell_{\widetilde{\rm OD3}})^2 \,. \tag{4.4}$$

Thus weakly coupled $\widetilde{\rm OD3}$ theory arises in the limit of large $\theta_{\rm YM}$ provided $g^2_{\rm YM}$ becomes small so that $\ell_{\widetilde{\rm OD3}}$ is kept fixed. At fixed energy scale E, this amounts to keeping $E\ell_{\widetilde{\rm OD3}}$ fixed while sending $G_{\widetilde{\rm OD3}} \to 0$. Thus $\widetilde{\rm OD3}$ theory and NCYM theory are related at weak (effective) coupling but in a limit where the perturbative description of the spatial non-commutativity in the NCYM theory breaks down. The tension of the NCYM 3-brane soliton is fixed in the limit and equal to the tension of the open D3-branes:

$$T_{\rm NC3s} \equiv \frac{1}{g^2_{\rm YM}\theta_{\rm YM}} = \frac{1}{\ell^4_{\widetilde{\rm OD3}}} \,. \tag{4.5}$$

Conversely, the NCYM theory arise in $\widetilde{\rm OD3}$ theory in the limit of small $G^2_{\widetilde{\rm OD3}}$ and small length scale, i.e. in the weakly coupled low energy limit, where it is natural to expect an effective field theory description.

A variant of the above reasoning shows that the $\widetilde{\rm OD3}$ theory may be a UV-completion of NCYM theory. We then keep $g^2_{\rm eff}$ fixed (instead of sending it to zero), by taking $g_{\rm YM}$ to zero and the energy scale E to infinity at fixed $\ell_{\widetilde{\rm OD3}}$. For large $E\ell_{\widetilde{\rm OD3}}$ and fixed $g^2_{\rm eff}$ we see that $G^2_{\widetilde{\rm OD3}}$ becomes small, which in turn implies that $\theta_{\rm eff}$ becomes large. Therefore the perturbative description of the NCYM theory breaks down, as expected, but since $\ell_{\widetilde{\rm OD3}}$ remains fixed and $G^2_{\widetilde{\rm OD3}}$ small, this points to a well-defined (finite) $\widetilde{\rm OD3}$ theory.

Thus, in summary we propose that OD3 and $\widetilde{\rm OD3}$ are S-dual UV-completions of six-dimensional NCYM theory (with non-commutativity in two spatial directions).

4.2 The $\widetilde{\rm OD}q$ theories ($q = 1, 2, 3$)

We expect the properties of the $\widetilde{\rm OD3}$ theory and its relation to NCYM theory to be invariant under T-duality, while the identification of its strong coupling duals of course changes. This leads to the notion of $\widetilde{\rm OD}q$ theories for $q = 1, 2, 3$ as $(q+3)$-dimensional non-gravitational theories containing light open Dq-branes that arise in the critical UV limits of the supergravity duals given by (2.14) (the case $q = 0$ is special, and will be discussed separately below). In section 3 we showed that the $\widetilde{\rm OD}q$ supergravity duals (2.14) are identical to the NCYM duals (2.8) upon the identification (3.1). It follows that the NCYM parameters (2.9) are related to the $\widetilde{\rm OD}q$ parameters in (2.15) as follows:[6]

$$g^2_{\rm YM} = (G^2_{\widetilde{{\rm OD}q}})^{\frac{2}{q+1}} \ell^{q-1}_{\widetilde{{\rm OD}q}} \,, \qquad \theta_{\rm YM} = (G^2_{\widetilde{{\rm OD}q}})^{\frac{-2}{q+1}} \ell^2_{\widetilde{{\rm OD}q}} \,. \tag{4.6}$$

[6] Only the relative strengths of the couplings are important; the precise values of the powers are of course just artifacts of the identification in (2.15). An important issue for future research is whether it is possible to give an independent derivation of covariant expressions for the open

The structure of these relations for general q is similar to that of $q=3$ given in (4.3). The important property is that $\ell_{\widetilde{\mathrm{OD}q}}$ remains fixed and $G^2_{\widetilde{\mathrm{ODq}}}$ becomes small in the limit of large θ_{YM} and small g^2_{YM}. Therefore the relations between NCYM theory and $\widetilde{\mathrm{OD3}}$ theory found in section 4.1, carry over to the other values of q. In particular, the tension of the NCYM solitons agrees with the tension of the light open Dq-branes, i.e. the relation (4.5) T-dualizes to $\theta_{\mathrm{YM}} g^2_{\mathrm{YM}} = \ell^{q+1}_{\widetilde{\mathrm{OD}q}}$.

The identification of $G^2_{\widetilde{\mathrm{OD}q}}$ in (2.15) can be verified independently in the case of $q=2$ by examining the uplifting of the $\widetilde{\mathrm{OD2}}$ theory supergravity dual, which yields the OM theory supergravity dual:

$$\begin{aligned}
\frac{ds^2_{11}}{\ell_p^2} &= \frac{1}{\ell^2_{\mathrm{OM}}}\left[\frac{\tilde r^2}{L^2}H^{1/3}\left(-d\tilde x_0^2 + d\tilde x_1^2 + d\tilde x_2^2\right) + \frac{L}{\tilde r}H^{-2/3}(d\tilde x_3^2 + d\tilde x_4^2 + d\tilde x_5^2) + \right.\\
&\qquad\qquad \left. + \frac{L}{\tilde r}H^{1/3}(d\tilde r^2 + \tilde r^2 d\Omega_4^2)\right],\\
\frac{A_{012}}{\ell_p^3} &= -\frac{1}{\ell^3_{\mathrm{OM}}}\frac{\tilde r^3}{L^3}, \qquad \frac{A_{345}}{\ell_p^3} = -\frac{1}{\ell^3_{\mathrm{OM}}}\frac{1}{H},\\
H &= 1 + \frac{L^3}{\tilde r^3}, \qquad L^3 = N\ell^3_{\mathrm{OM}}, \qquad\qquad (4.7)
\end{aligned}$$

provided we make the following identification of parameters:

$$R_{\mathrm{m}} = G^{4/3}_{\widetilde{\mathrm{OD2}}}\ell_{\widetilde{\mathrm{OD2}}}, \qquad \ell_{\mathrm{OM}} = \ell_{\widetilde{\mathrm{OD2}}}. \qquad (4.8)$$

Hence the radius R_{m} of the magnetic circle on which the $\widetilde{\mathrm{OD2}}$ theory descends from OM theory indeed becomes small in units of $\ell_{\widetilde{\mathrm{OD2}}}$ when $G^2_{\widetilde{\mathrm{OD2}}}$ becomes small.

Interestingly, it follows from (4.8) that OM theory can descend to weakly coupled $\widetilde{\mathrm{OD2}}$ theory on a small circle while the effective OM coupling $E\ell_{\mathrm{OM}}$, where E is the six-dimensional energy scale, remains finite. This suggests that the OM theory interactions are reminiscent of those in the weakly coupled $\widetilde{\mathrm{OD2}}$ theory.

The strong coupling limit of the $\widetilde{\mathrm{OD1}}$ theory will be discussed in section 5.

4.3 The $\widetilde{\mathrm{OD0}}$ theory

The three-dimensional $\widetilde{\mathrm{OD0}}$ theory arises in the IR limit of the supergravity dual (2.14) and is dual to NCYM theory and the SO(8)-invariant conformal theory on the M-theory membrane. The lift of the $\widetilde{\mathrm{OD0}}$ supergravity dual (2.14) to M-theory gives the near horizon region of a M2-MW bound state (the membrane is transverse to the wave but smeared in the 11 direction):

$$\frac{ds^2}{\ell_p^2} = \frac{1}{\ell^2_{\mathrm{Pl}}}H^{1/3}\left(\frac{\tilde r}{\tilde R}\right)^5\left[-d\tilde x_0^2 + H^{-1}\left(\frac{\tilde R}{\tilde r}\right)^5(d\tilde x_1^2 + d\tilde x_2^2) + \left(\frac{\tilde R}{\tilde r}\right)^5(d\tilde r^2 + \tilde r^2 d\Omega_6^2) + \right.$$

D-brane coupling and tension, which are valid in an arbitrary slowly varying background, that will justify (2.15).

$$+ H^{-1}\left(\frac{\tilde R}{\tilde r}\right)^{10}\left(d\tilde x^{11}+\left(\frac{\tilde r}{\tilde R}\right)^5 d\tilde x^0\right)^2\Bigg],$$

$$\frac{A_{012}}{\ell_p^3}=\frac{1}{\ell_{\rm Pl}^3}\frac{1}{H}\,,\qquad \frac{A_{12,11}}{\ell_p^3}=-\frac{1}{\ell_{\rm Pl}^3}\frac{1}{H}\,,\qquad H=1+\left(\frac{\tilde R}{\tilde r}\right)^5,\qquad \tilde R^5=\frac{N\ell_{\rm Pl}^6}{R_{11}}\,,\tag{4.9}$$

with eleven-dimensional length scale $\ell_{\rm Pl}$ and radius R_{11} given by:

$$\ell_{\rm Pl}=g^{1/3}\tilde\ell=G_{\widetilde{\rm OD0}}^{-4/3}\ell_{\widetilde{\rm OD0}}\,,\qquad R_{11}=g\tilde\ell=\ell_{\widetilde{\rm OD0}}\,.\tag{4.10}$$

For large $\tilde r/\tilde R$ the eleven-dimensional geometry is dominated by a wave propagating in the light-like 11 direction in a flat background with Planck length $\ell_{\rm Pl}$. The ten-dimensional UV region is dominated by the D0-brane charge. In the gauge $X^0=\tau$ the D0-brane action reduces in the UV limit to the quantum mechanical kinetic term $\frac{1}{2}M_{\widetilde{\rm OD0}}\dot X^2$ where $\dot X$ is the nine-dimensional velocity and the D0-mass is given by

$$M_{\widetilde{\rm OD0}}=(\ell_{\widetilde{\rm OD0}})^{-1}\,.\tag{4.11}$$

This is the supergravity dual description [30] (the BGLHKS limit) of the original DLCQ limit of M-theory [31, 32] (the DKPS and SSS limits).[7] Thus the UV limit of the $\widetilde{\rm OD0}$ theory with N units of D0-brane charge is a DLCQ compactification of M-theory with N units of DLCQ momentum on a light-like circle with radius R_{11}, in the presence of transverse M2-branes. This is similar to the strong coupling limit of the OD0 theory [1] (change the M2-brane to an M5-brane). However, unlike the usual undeformed matrix model which have only a dimensionful coupling, the $\widetilde{\rm OD0}$ matrix model inherits the dimensionless coupling $G^2_{\widetilde{\rm OD0}}$, which is encoded in the transverse two-brane. From (4.10) it follows that $\ell_{\rm Pl}$ becomes large when $G^2_{\widetilde{\rm OD0}}\ll 1$ and $\ell_{\widetilde{\rm OD0}}$ is fixed, while it becomes small, leading to eleven-dimensional supergravity, at strong $\widetilde{\rm OD0}$ coupling, $G^2_{\widetilde{\rm OD0}}\gg 1$, and fixed $\ell_{\widetilde{\rm OD0}}$. In the limit of large N, $G^2_{\widetilde{\rm OD0}}$ and $\ell_{\widetilde{\rm OD0}}$ (at fixed energy) we recover the BFSS limit, i.e. $R_{11}\to\infty$ at fixed $\ell_{\rm Pl}$ and light-like momentum N/R_{11}, which leads to weakly coupled supergravity with finite but small derivative corrections.

For small $\tilde r/\tilde R$, the geometry is dominated by the two-branes, so that the open D0-brane physics becomes three-dimensional, instead of strongly coupled quantum mechanics. In the open string picture this gives rise to three-dimensional NCYM theory with fixed $g^2_{\rm YM}$ and $\theta_{\rm YM}$ given by (4.6) (the non-commutativity is independent of $\tilde r/\tilde R$ and non zero in the IR even though the NS potential vanishes in there). Its strong coupling dual is the SO(8)-invariant conformal theory (the $N=8$ singleton) on the M2-brane [1]. This theory has a critical mass-term determined by the length scale $\ell_{\rm PL}$ of the supergravity dual (4.9). Dualizing one of its scalars, by first identifying it with a circle with radius R_{11}, leads to a NCYM theory with supergravity dual (2.8)

[7] For a review see e.g. [33].

provided that:

$$\ell_{\mathrm{SO}(8)} \equiv \ell_{\mathrm{Pl}} = g_{\mathrm{YM}}^{2/3}\theta_{\mathrm{YM}}^{2/3}\,, \qquad R_{11} = g_{\mathrm{YM}}^{2}\theta_{\mathrm{YM}}\,. \tag{4.12}$$

Thus at strong coupling, $g_{\mathrm{YM}}^2 \gg E$ where E is a fixed energy, and small θ_{YM}, such that $g_{\mathrm{YM}}\theta_{\mathrm{YM}}$ is fixed, the NCYM theory goes over into the SO(8) theory with large R_{11} and fixed $\ell_{\mathrm{SO}(8)}$, that is fixed effective coupling $E\ell_{\mathrm{SO}(8)}$.

In the limit of large θ_{YM}, small g_{YM}^2 and fixed NCYM soliton mass $(g_{\mathrm{YM}}^2\theta_{\mathrm{YM}})^{-1}$ both NCYM and SO(8) theories break down, and the perturbative description is given by the $\widetilde{\mathrm{OD0}}$ theory which has an expansion in $G^2_{\widetilde{\mathrm{OD0}}}$ at fixed $\ell_{\widetilde{\mathrm{OD0}}}$. The NCYM solitons remain weakly coupled, provided the strong NCYM interactions at long wave-length are under control, and their mass is equal to the D0-mass given in (4.11), which is constant under the flow from UV down to the IR. We propose that the $\widetilde{\mathrm{OD0}}$ theory is based on critical open D0-branes, which are identified as D0-branes that are created on the D2-brane and remains bound to it (in a weak, sub-critical electric field). The strong coupling dual of the $\widetilde{\mathrm{OD0}}$ theory is the SO(8) theory. For small $G^2_{\widetilde{\mathrm{OD0}}}$ and fixed $\ell_{\widetilde{\mathrm{OD0}}}$ the SO(8) theory becomes strongly coupled. On the other hand, for large $G^2_{\widetilde{\mathrm{OD0}}}$ and fixed $\ell_{\widetilde{\mathrm{OD0}}}$ the SO(8) length scale $\ell_{\mathrm{SO}(8)}$ becomes small and R_{11} fixed, while if $\ell_{\widetilde{\mathrm{OD0}}}$ is taken to be large (in units of a fixed energy) then $\ell_{\mathrm{SO}(8)}$ is fixed and R_{11} becomes large.

5. Non-commutative generalized gauge and tensor theories on D-branes and NS5-branes

In the previous section we examined a new type of RR-electric theories on D-branes, the $\widetilde{\mathrm{OD}q}$ theories, and found that they are related to NCYM theories at weak coupling and large θ_{YM} and to other theories at large coupling and fixed length scale. In this section we show that the arrangement of the NCOS, NCYM, ODq and $\widetilde{\mathrm{OD}q}$ theories into duality webs requires another set of decoupled theories — the Dq-GT and the Dq-$\widetilde{\mathrm{GT}}$ — that are perturbative in the corners of the parameter space corresponding to the supergravity duals (2.17) and (2.21) with fixed parameters (2.18) and (2.22), respectively. These supergravity solutions are obtained in magnetic near horizon limits of D-branes and NS5-branes with fixed magnetic RR fluxes in the UV, and as was shown in section 3 they are equivalent to the NCOS and ODp duals (2.6) and (2.11). This means that the new theories have fixed spatial non-commutativity in limits where the temporal non-commutativity length scales of the NCOS and ODp theories become large.

5.1 Generalized gauge theories on D-branes (Dq-GT)

The Dq-GT is defined by the supergravity dual (2.17) and has coupling $g^2_{\mathrm{D}q\text{-GT}}$ and spatial non-commutativity parameter $\theta_{\mathrm{D}q\text{-GT}}$, which measures the strength of an anti-symmetric tensor of rank $(q+1)$ and of dimension length to the power $q+1$, given in (2.18). From (2.7) and (3.1) it follows that:

$$g^2_{\mathrm{D}q\text{-GT}} = G^2_{\mathrm{OS}}(\alpha'_{\mathrm{eff}})^{\frac{q-1}{2}}\,, \qquad \theta_{\mathrm{D}q\text{-GT}} = G^2_{\mathrm{OS}}(\alpha'_{\mathrm{eff}})^{\frac{q+1}{2}}\,. \tag{5.1}$$

This relation together with the assumption that the NCOS theory should be related to Dq-GT at weak coupling implies that the Dq-GT (if it exists) must arise in the $\alpha'_{\mathrm{eff}} \to \infty$, or high energy, limit of NCOS at weak coupling $G^2_{\mathrm{OS}} \to 0$ such that $\theta_{\mathrm{D}q\text{-GT}}$ is kept fixed. In this limit the SYM sector of the NCOS theory, whose Yang-Mills coupling is given by

$$g^2_{\mathrm{YM}} \equiv G^2_{\mathrm{OS}}(\alpha'_{\mathrm{eff}})^{\frac{q-1}{2}} = g^2_{\mathrm{D}q\text{-GT}}\,, \tag{5.2}$$

therefore remains interacting, which motivates the term "generalized gauge theory".

The perturbative formulation of ordinary (commutative) open string theory is not known in this limit, and one may argue that it simply consists of some kind of degenerate open string world sheets. However, we believe that the NCOS theory may stand a better chance, because the supergravity dual description indicates that the theory could be described in terms of some structure which encodes the fixed non-commutativity parameter $\theta_{\mathrm{D}q\text{-GT}}$. For $q = 2, 3$ this type of non-commutativity is naturally represented on $(q-1)$-loop space; see for example [34] for a similar discussion in the case of the M-theory membrane. This is indicative of a non-commutative generalization of the ordinary loop space covariant derivatives. A preliminary analysis hints that such a structure indeed accommodates a hierarchy of massless gauge fields in $q+3$ dimensions, as one might anticipate in the above limit. If we let G_{loop} be the gauge coupling and assume that at very low energies the resulting theory contains a SYM sector, then a simple-dimensional analysis shows that $g^2_{\mathrm{YM}} = G^2_{\mathrm{loop}}(\theta_{\mathrm{D}q\text{-GT}})^{\frac{q-1}{q+1}}$. From (5.2) it then follows that $G^2_{\mathrm{loop}} = (G^2_{\mathrm{OS}})^{\frac{2}{q+1}}$ so that G_{loop} is indeed small in the limit when G^2_{OS} is small.

For $q = 2, 3$ the Dq-GT coupling is dimensionful and we shall assume that it sets the energy scale of the Dq-GT. From (5.1) it follows that if the Dq-GT non-commutativity scale, set by the parameter $\theta_{\mathrm{D}q\text{-GT}}$, is kept fixed in the limit of large α'_{eff} and small G^2_{OS} at fixed energy E, then the effective Dq-GT coupling $g^2_{\mathrm{D}q\text{-GT}}E^{q-1}$ becomes small. In particular, this means that the effects of the non-commutativity become visible at energies where the effective Dq-GT coupling still is small. Thus the Dq-GT is expected to have a formulation at long wave-lengths in terms of some weakly coupled field theory construction, such as the loop-space gauge theory discussed above.

The case $q = 0$ appears to be special. It is not clear to us how to interpret the vectorial non-commutativity parameter in terms of a non-commutative structure on some space. We shall therefore simply define the D0-GT as the large α'_{eff} and small G^2_{OS} limit of three-dimensional NCOS theory.

JHEP06(2001)008

5.2 Extended duality webs

We next turn to the issue of identifying the strong coupling limits of the Dq-GT and arrange it together with the other non-commutative theories into duality diagrams involving limits of strong coupling as well as large non-commutativity length scale.

5.2.1 Four-dimensional non-commutative theories

We have four different four-dimensional non-commutative theories on the D3-brane: the NCYM and NCOS theories based on weakly coupled open F1-string interactions at fixed non-commutativity length scale; the $\widetilde{\mathrm{OD1}}$ theory which has a $G^2_{\widetilde{\mathrm{OD1}}} = g^2_{\mathrm{YM}}$ expansion at fixed $\ell^2_{\widetilde{\mathrm{OD1}}} = \theta_{\mathrm{YM}} g^2_{\mathrm{YM}}$; and the D1-GT which has a G^2_{OS}-expansion at fixed $\theta_{\text{D1-GT}} = \alpha'_{\mathrm{eff}} G^2_{\mathrm{OS}}$. Upon combining the relations (4.6) and (5.1) with the usual S-duality relations between NCOS and NCYM theory, which read

$$g^2_{\mathrm{YM}} = \frac{1}{G^2_{\mathrm{OS}}}\,, \qquad \theta_{\mathrm{YM}} = G^2_{\mathrm{OS}} \alpha'_{\mathrm{eff}}\,, \tag{5.3}$$

we find that at strong coupling and fixed length scales, the NCOS theory go over into $\widetilde{\mathrm{OD1}}$ theory, as follows from

$$G^2_{\mathrm{OS}} = \frac{1}{G^2_{\widetilde{\mathrm{OD1}}}}\,, \qquad \alpha'_{\mathrm{eff}} = \ell^2_{\widetilde{\mathrm{OD1}}}\,, \tag{5.4}$$

and NCYM theory into D1-GT, as follows from:

$$g^2_{\mathrm{YM}} = \frac{1}{g^2_{\text{D1-GT}}}\,, \qquad \theta_{\mathrm{YM}} = \theta_{\text{D1-GT}}\,. \tag{5.5}$$

In going from NCYM theory to D1-GT theory the non-commutativity parameter is held fixed so that the NCYM solitons become massless, which is equivalent with the D1-GT being the tensionless limit of the NCOS theory. Of course, NCOS go over into NCYM theory in the limit of strong coupling and small α'_{eff} such that θ_{YM} is fixed [12]. This can be summarized in the following duality diagram:

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\widetilde{\mathrm{OD1}} & \mathrm{NCYM} \arrow[l, "\substack{\theta_{\mathrm{YM}} \to \infty \\ \theta_{\mathrm{YM}} g^2_{\mathrm{YM}}\ \text{fixed}}"'] \arrow[d, "\substack{g^2_{\mathrm{YM}} \to \infty \\ \theta_{\mathrm{YM}}\ \text{fixed}}"] \\
\mathrm{NCOS} \arrow[u, "\substack{G^2_{\mathrm{OS}} \to \infty \\ \alpha'_{\mathrm{eff}}\ \text{fixed}}"] \arrow[r, "\substack{\alpha'_{\mathrm{eff}} \to \infty \\ \alpha'_{\mathrm{eff}} G^2_{\mathrm{OS}}\ \text{fixed}}"'] & \text{D1-GT}
\end{tikzcd}$$

There is an important asymmetry in the diagram, so that both types of strings are heavy in NCYM theory, whereas one type is light and the other one heavy in D1-GT. Moreover, as already mentioned in the introduction, the IIB $\mathrm{SL}(2,\mathbb{Z})$ does not map the weakly coupled supergravity duals of the lower part of the diagram into the the weakly coupled supergravity duals of the upper part of the diagram, so that it would indeed be artificial to assume that the diagram would be self-dual in the sense that the lower half could be identified with the upper half (of course, when the coupling becomes large in the lower part of the diagram, then the IIB $\mathrm{SL}(2,\mathbb{Z})$ maps it to the upper part and vice versa).

5.2.2 Five-dimensional non-commutative theories

In five dimensions we can organize the non-commutative theories as follows:

$$
\begin{tikzcd}
\widetilde{\mathrm{OD2}} & & \mathrm{NCYM} \arrow[ll, "\substack{\theta_{\mathrm{YM}} \to \infty \\ \theta_{\mathrm{YM}} g^2_{\mathrm{YM}} \text{ fixed}}"'] \arrow[dl, dash, "S^1_{\mathrm{m}}"] \\
& \mathrm{OM} \arrow[ul, dash, "S^1_{\mathrm{m}}"'] \arrow[dl, dash, "S^1_{\mathrm{e}}"'] \arrow[dr, dash, "S^1_{\mathrm{e}}"] & \\
\mathrm{NCOS} \arrow[rr, "\substack{\alpha'_{\mathrm{eff}} \to \infty \\ \alpha'_{\mathrm{eff}} G^{4/3}_{\mathrm{OS}} \text{ fixed}}"'] & & \text{D2-GT}
\end{tikzcd}
$$

Here S^1_{m} and S^1_{e} denote electric and magnetic circles in OM theory. The relations between the parameters of the electric reduction of OM theory and the ones of NCOS theory and D2-GT are given by

$$
\begin{aligned}
\ell_{\mathrm{OM}} &= G^{2/3}_{\mathrm{OS}} \sqrt{\alpha'_{\mathrm{eff}}}\,, \\
R_{\mathrm{e}} &= G^2_{\mathrm{OS}} \sqrt{\alpha'_{\mathrm{eff}}}\,, \\
\ell_{\mathrm{OM}} &= \theta^{1/3}_{\text{D2-GT}}\,, \\
R_e &= g^2_{\text{D2-GT}}\,.
\end{aligned}
\tag{5.6}
$$

The relations in the magnetic case follow from (4.6) and (4.8), which we collect here for completeness:

$$
\begin{aligned}
\ell_{\rm OM} &= g_{\rm YM}^{2/3}\theta_{\rm YM}^{1/3}\,, \qquad R_{\rm m} = g_{\rm YM}^2\,, \\
\ell_{\rm OM} &= \ell_{\widetilde{\rm OD2}}\,, \qquad R_{\rm m} = G_{\widetilde{\rm OD2}}^{4/3}\ell_{\widetilde{\rm OD2}}\,.
\end{aligned} \tag{5.7}
$$

Thus the reductions of OM theory at fixed energy E to D2-GT and $\widetilde{\rm OD2}$ keeps the six-dimensional effective coupling $g^2_{\rm OM,eff} \equiv E\ell_{\rm OM}$ fixed, as opposed to the reductions to the NCOS and NCYM theories where $g^2_{\rm OM,eff} \to 0$ at weak string coupling.

5.2.3 Three-dimensional non-commutative theories

In [1] it was conjectured that three-dimensional NCOS theory lifts to the M2-brane SO(8) theory in the limit of strong coupling. The D0-GT theory has the same supergravity dual as the NCOS theory and obey the relation (5.1) with $q = 0$. We therefore conjecture that the D0-GT theory has the same strong coupling limit as three-dimensional NCOS theory. The SO(8) theory also governs the strong coupling behavior of three-dimensional NCYM theory. In section 4.3 we proposed to extend this to the $\widetilde{\rm OD0}$ theory. Thus we organize the non-commutative theories in three dimensions as follows:

$$
\begin{tikzcd}
\widetilde{\rm OD0} & & {\rm NCYM} \arrow[ll, "\theta_{\rm YM}\to\infty"'] \arrow[dl, dash, "S^1_{\rm W}"] \\
& {\rm SO(8)} \arrow[ul, dash, "S^1_{\rm W}"] \arrow[dl, dash, "S^1_{\rm M2}"'] \arrow[dr, dash, "S^1_{\rm M2}"] & \\
{\rm NCOS} \arrow[rr, "\alpha'_{\rm eff}\to\infty"'] & & {\rm D0\text{-}GT}
\end{tikzcd}
$$

Here S^1_W amounts to a boost of the M2-brane followed by a transverse reduction in the resulting wave direction. $S^1_{\rm M2}$ denotes a reduction along a (skew) direction in a smeared M2-M2 bound state. The relations between the SO(8) theory parameters and the non-commutative theories in type IIA are given by (4.10) and (4.12). As in the five-dimensional case we find that the reductions to the D0-brane theories keep the effective SO(8) coupling $E\ell_{\rm SO(8)}$ fixed at fixed energy E.

5.3 Generalized gauge theories on the type-IIB NS5-brane

On the IIB five-branes the situation is not yet quite symmetric; after the inclusion of the $\widetilde{\text{OD3}}$ theory and the D3-GT we have two "triplets": i) 1) NCOS theory (NS-electric D5 in the open F1-picture); 2) D3-GT (RR-magnetic D5-brane in the open D3-brane picture); and 3) OD1 theory (RR-electric IIB NS5-brane in LST-picture), and ii) 1') NCYM theory (NS-magnetic D5-brane in the open F1-picture); 2') $\widetilde{\text{OD3}}$ theory (RR-electric D5-brane in the open D3-brane picture); and 3') OD3 theory (RR-electric IIB NS5-brane in the LST-picture).

The triplet (i) is completed by a theory, which we shall denote by D3-$\widetilde{\text{GT}}$, that arises in the open D3-brane picture on a RR-magnetic NS5-brane. We shall assume that D3-$\widetilde{\text{GT}}$ is some limit of OD1 at weak coupling $G^2_{\text{OD1}} \ll 1$. From (2.22) and (3.2) it follows that the D3-$\widetilde{\text{GT}}$ theory has coupling (with dimension energy to the power two) and spatial non-commutativity parameter (which is an anti-symmetric tensor of rank 4 and dimension length to the power 4) given by

$$g^2_{\text{D3-}\widetilde{\text{GT}}} = \frac{1}{\ell^2_{\text{LST}}}\,, \qquad \theta_{\text{D3-}\widetilde{\text{GT}}} = G^2_{\text{OD1}}\ell^4_{\text{LST}}\,. \tag{5.8}$$

Hence the D3-$\widetilde{\text{GT}}$ arises in the $\ell_{\text{LST}} \to \infty$ limit of OD1, at weak OD1 coupling and fixed $\theta_{\text{D3-}\widetilde{\text{GT}}}$. Importantly, the OD1 solitons become heavy in this limit, since their length scale $G_{\text{OD1}}\ell_{\text{LST}}$ becomes small. The positive dimension of the coupling means that the D3-$\widetilde{\text{GT}}$ becomes strongly coupled at energies below $g_{\text{D3-}\widetilde{\text{GT}}} = \ell^{-1}_{\text{LST}}$, which is due to little string effects. Thus, for energies above ℓ^{-1}_{LST} we expect to describe D3-$\widetilde{\text{GT}}$ in some language free of little strings. Hence, for large ℓ_{LST}, small G^2_{OD1} and fixed $\theta_{\text{D3-}\widetilde{\text{GT}}}$ there is an interval of energies $g_{\text{D3-}\widetilde{\text{GT}}} \ll E \ll \theta^{-1/4}_{\text{D3-}\widetilde{\text{GT}}}$ where the little string effects are unimportant and the effective non-commutativity $E^4\theta_{\text{D3-}\widetilde{\text{GT}}}$ is small, so that it can be described perturbatively using some generalized gauge theory based on open D3-branes. Thus D3-$\widetilde{\text{GT}}$ differs from OD1 theory, whose temporal non-commutativity becomes visible first at strong effective coupling at energies above the little string scale.

We therefore expect the structure of the D3-$\widetilde{\text{GT}}$ to resemble that of the D3-GT. Indeed, their supergravity duals, (2.17) and (2.21), are related by S-duality without any change in the magnetic RR 4-form potential, which implies

$$\theta_{\text{D3-}\widetilde{\text{GT}}} = \theta_{\text{D3-GT}}\,, \qquad g^2_{\text{D3-}\widetilde{\text{GT}}} = \frac{1}{g^2_{\text{D3-GT}}}\,. \tag{5.9}$$

In order to interpret this, we recall that the generalized gauge theory description of D3-GT is perturbative when $E \ll \theta^{-1/4}_{\text{D3-GT}} \ll g^{-1}_{\text{D3-GT}}$. From (5.9 it follows that when the generalized gauge theory description of D3-GT breaks down at $\theta^{-1/4}_{\text{D3-GT}} > g^{-1}_{\text{D3-GT}}$ then the corresponding formulation of D3-$\widetilde{\text{GT}}$ becomes well defined. The generalized gauge theory descriptions of D3-GT and D3-$\widetilde{\mathrm{GT}}$ are therefore S-dual at fixed energy and non-commutativity parameter. We summarize as follows:

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\mathrm{OD1} \arrow[r, "{\begin{matrix}\ell_{\mathrm{LST}}\to\infty \\ \ell^4_{\mathrm{LST}}G^2_{\widetilde{\mathrm{OD1}}}\ \text{fixed}\end{matrix}}"] \arrow[d, dash, "\mathrm{S}"'] & \mathrm{D3}\text{-}\widetilde{\mathrm{GT}} \arrow[d, dash, "\mathrm{S}"] \\
\mathrm{NCOS} \arrow[r, "{\begin{matrix}\alpha'_{\mathrm{eff}}\to\infty \\ (\alpha'_{\mathrm{eff}})^2G^2_{\mathrm{OS}}\ \text{fixed}\end{matrix}}"'] & \mathrm{D3}\text{-}\mathrm{GT}
\end{tikzcd}$$

Similarly, the triplet (ii) is completed by D1-$\widetilde{\mathrm{GT}}$ that is S-dual to NCYM theory at fixed length and the tensionless little string limit of OD3 at weak OD3-coupling:

$$\begin{aligned}
\theta_{\mathrm{D1}\text{-}\widetilde{\mathrm{GT}}} &= \theta_{\mathrm{YM}} = G^2_{\mathrm{OD3}}\ell^2_{\mathrm{LST}}\,, \\
g^2_{\mathrm{D1}\text{-}\widetilde{\mathrm{GT}}} &= \frac{1}{g^2_{\mathrm{YM}}} = \frac{1}{\ell^2_{\mathrm{LST}}}\,.
\end{aligned} \tag{5.10}$$

The reason D1-$\widetilde{\mathrm{GT}}$ is S-dual to NCYM theory, and not another generalized gauge theory as was the case for D3-$\widetilde{\mathrm{GT}}$, is related to that the solitonic D3-branes become massless in the tensionless little string limit of OD3. This is equivalent to the vanishing of the tension (4.5) of the non-commutative solitons in the six-dimensional NCYM theory in the limit of large g^2_{YM} and fixed θ_{YM}. Thus, from (5.10) it follows that we can identify D1-$\widetilde{\mathrm{GT}}$ with six-dimensional NCYM theory in the limit of large g^2_{YM} at fixed θ_{YM}. We summarize as follows:

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\mathrm{OD3} \arrow[r, "{\begin{matrix}\ell_{\mathrm{LST}}\to\infty \\ \ell^2_{\mathrm{LST}}G^2_{\mathrm{OD3}}\ \text{fixed}\end{matrix}}"] \arrow[d, dash, "\mathrm{S}"'] & \mathrm{D1}\text{-}\widetilde{\mathrm{GT}} \arrow[d, dash, "\mathrm{S}"] \\
\widetilde{\mathrm{OD3}} & \mathrm{NCYM} \arrow[l, "{\begin{matrix}\theta_{\mathrm{YM}}\to\infty \\ \theta_{\mathrm{YM}}g^2_{\mathrm{YM}}\ \text{fixed}\end{matrix}}"]
\end{tikzcd}$$

5.4 Generalized tensor theories on the type-IIA NS5-brane

T-dualizing the Dq-GT ($q = 1, 3$) on the IIB NS5-brane leads to the Dq-$\widetilde{\mathrm{GT}}$ ($q = 0, 2, 4$) on the IIA NS5-brane based on the supergravity duals (2.21). These theories have spatial non-commutativity and coupling with positive energy dimension given by (2.18). For energies larger than the coupling and smaller than the non-commutative length scale we expect these theories to have perturbative formulations in terms of some tensionless extensions of the $(2, 0)$ tensor theory incorporating loop space non-commutativity.

The supergravity duals of Dq-$\widetilde{\mathrm{GT}}$ and OD$(4-q)$ theory are related, as shown in section 3, leading to relations between their parameters analogous to (5.8). In the limit of large ℓ_{LST} and fixed $\theta_{\mathrm{D}q\text{-}\widetilde{\mathrm{GT}}}$ the solitonic D$(4-q)$-branes of the OD$(4-q)$ theory become tensionless when $q = 0$ and heavy when $q = 4$, while the solitonic D2-branes of the OD2 theory remain in the spectrum with a finite tension given by the non-commutative length scale:

$$T_{\mathrm{OD2}} = \frac{1}{\tilde{g}\tilde{\ell}^3} = \frac{1}{\theta_{\mathrm{D2}\text{-}\widetilde{\mathrm{GT}}}}\,, \tag{5.11}$$

where we have used (2.12), (2.22) and (3.2). The generalized tensor theory becomes strongly coupled when $g_{\mathrm{D}q\text{-}\widetilde{\mathrm{GT}}}\theta^{\frac{1}{q+1}}_{\mathrm{D}q\text{-}\widetilde{\mathrm{GT}}} > 1$, since then there is no energy interval where both little string and non-commutative effects are weak.

The supergravity dual of D0-$\widetilde{\mathrm{GT}}$ and the OD4 theory is a reduction of the near horizon region of a (smeared) M5-M5 bound state along a skew direction. We therefore expect these theories to be governed at strong coupling by the M5-brane $(2, 0)$ tensor multiplet. The supergravity duals of D4-$\widetilde{\mathrm{GT}}$ and the OD0 theory descend from a boosted M5-brane with a transverse wave. The wave provides the electric D0-brane charge, which becomes heavy in the limit of small radius.

The case of $q = 2$ requires extra care. The six-dimensional worldvolume has self-dual RR three-form flux. The electric near horizon limit yields the OD2 supergravity dual (2.11) and the magnetic near horizon limit yields the D2-$\widetilde{\mathrm{GT}}$ supergravity dual (2.21). These are equivalent,[8] as we have shown in section 3. Thus the OD2 and D2-$\widetilde{\mathrm{GT}}$ theories are perturbative descriptions valid for small and large ℓ_{LST}, respectively, at fixed energy (for large ℓ_{LST} we take small dimensionless coupling so that $\theta_{\mathrm{D2}\text{-}\widetilde{\mathrm{GT}}}$ is fixed). For energies $g_{\mathrm{D2}\text{-}\widetilde{\mathrm{GT}}} \ll E \ll \theta^{-1/3}_{\mathrm{OD2}} = \ell^{-1}_{\mathrm{OD2}}$ the D2-$\widetilde{\mathrm{GT}}$ theory is expected to have a perturbative expansion as a generalized tensor theory with weak spatial non-commutativity. When $g_{\mathrm{D2}\text{-}\widetilde{\mathrm{GT}}} > \theta^{-1/3}_{\mathrm{OD2}}$ the theory becomes strongly

[8] Thus, if we were to use an open D2-brane description, their fluctuations around the NS5-brane would form a "mixed state" of fluctuations in both electric and magnetic directions, similar to the (p, q)-string fluctuations in the IIB theory. The perturbative description of the OD2 theory and the D2-$\widetilde{\mathrm{GT}}$ would then have to be extracted from a single sector treated as fundamental, say in a semi-classical expansion around a classical solution.

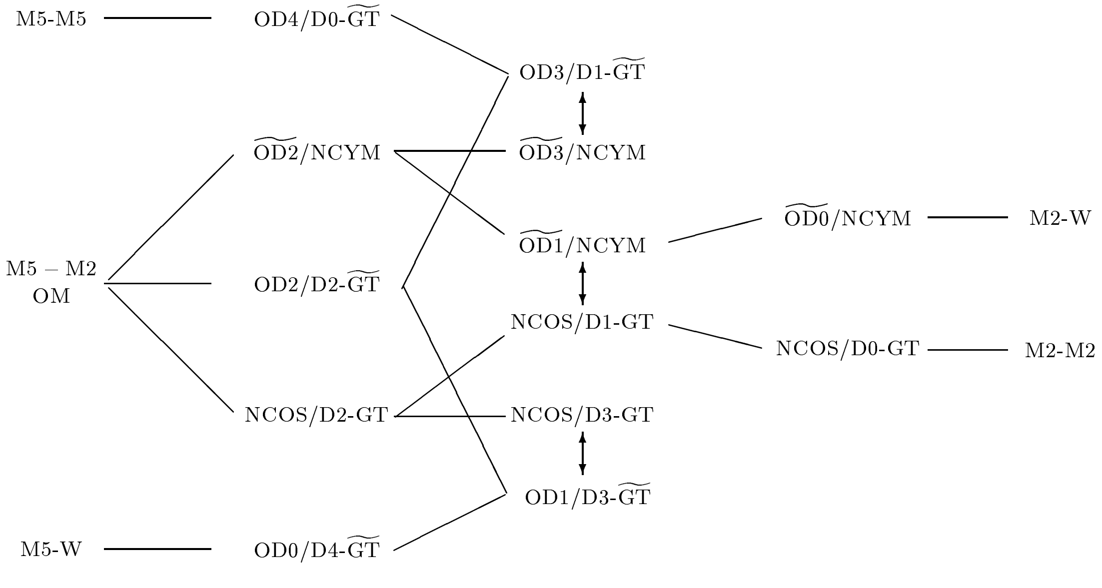
coupled. The uplift of the OD2 and D2-$\widetilde{\mathrm{GT}}$ supergravity duals is identified with the OM theory supergravity dual reduced on a transverse circle S^1_T [4]:

$$\begin{aligned} \ell_{\mathrm{OM}} &= G^{2/3}_{\mathrm{OD2}}\ell_{\mathrm{LST}} = \theta^{1/3}_{\mathrm{D2}\text{-}\widetilde{\mathrm{GT}}}\,, \\ R_T &= G^{2}_{\mathrm{OD2}}\ell_{\mathrm{LST}} = g^{2}_{\mathrm{D2}\text{-}\widetilde{\mathrm{GT}}}\theta_{\mathrm{D2}\text{-}\widetilde{\mathrm{GT}}}\,. \end{aligned} \tag{5.12}$$

This means that OM theory at a fixed energy E and fixed effective coupling $E\ell_{\mathrm{OM}}$ wrapped on a circle of small radius $ER \ll 1$ can be described in terms of weakly coupled D2-$\widetilde{\mathrm{GT}}$ theory.

6. Summary and discussion

In this paper we have examined some implications of the fact that a given bound state solution may be viewed either as an electric deformation or as a magnetic deformation, and that the corresponding near horizon limits lead to equivalent near horizon regions. We propose that a new class of open D-brane theories, the $\widetilde{\mathrm{OD}q}$ theories, arise on D-branes with critical electric RR fluxes, and that they are S-dual to NCOS or ODp theories. We have also argued that the $\widetilde{\mathrm{OD}q}$ theories are related to NCYM theories in limits of weak coupling involving NCYM solitons of fixed tension. We then constructed duality webs, involving limits of both strong coupling and large non-commutativity scale, and showed that these require new theories with fixed spatial non-commutativity, the Dq-GT on D-branes and Dq-$\widetilde{\mathrm{GT}}$ on NS5-branes, that arise as perturbative descriptions in the tensionless open and little string limits of NCOS and ODp theory, respectively, at weak open string coupling or ODp-coupling. An attempt to unify the duality diagrams for the non-commutative theories assume the following shape:



In this figure, in the first and fifth column we have theories which originates from M-theory, while the theories in column two and four originate from type IIA and column three from type-IIB string theory. The horizontal and diagonal lines indicate T-dualities between IIA/B theory or lifts/reductions between IIA/M-theory. The vertical lines imply S-dualities between theories in IIB. We remark that they follow, in the usual way, from the proposed modular invariance of M-theory/OM theory [35, 2]. For example, dropping on a transverse circle from OM theory to OD2/D2-$\widetilde{\mathrm{GT}}$, and then performing an electric T-duality (i.e. a T-duality along a direction with electric field) leads to OD1/D3-$\widetilde{\mathrm{GT}}$. We can also reduce OM theory on an electric circle to NCOS/D2-GT and then perform a T-duality along a transverse direction, which leads to NCOS/D3-GT. The S-duality between these two pairs (which are internally related at small and large energies) now follows by performing the modular transformation of the electric torus. Similar arguments show the remaining two S-dualities in IIB from the modular transformations of a magnetic torus and a magnetic/electric torus.

The results in this paper points to the possibility of describing D-brane dynamics using open D-branes rather than open strings in weak coupling limits where the non-commutativity in the open string sector becomes large. Let us end this discussion with some speculations along these lines.

For NCYM (large spatial non-commutativity) this limit involves NCYM solitons with fixed tension, which describe sub-critical open D-branes as long as θ_{YM} is finite. The angle between the open D-brane and the D-brane on which it ends is then non zero, so there are no additional massless open string modes in the spectrum. The $\widetilde{\mathrm{OD}q}$ theories, on the other hand, are by definition assumed to have perturbative expansions in their couplings $G^2_{\widetilde{\mathrm{ODq}}}$ at fixed $\theta_{\widetilde{\mathrm{OD}q}} = \theta_{\mathrm{YM}} g^2_{\mathrm{YM}}$. This formulation is thus expected to involve critical open D-branes at zero angle. At this critical angle the perturbative spectrum changes abruptly, due to the new massless open string modes propagating along the open D-brane. In particular, in the case of open D-strings it is therefore reasonable to assume that the interactions between the critical open D-strings are not the ordinary fundamental self-interactions but rather interactions of a new type, which are related to the other $\widetilde{\mathrm{OD}q}$ interactions by T-duality.

For NCOS (large temporal non-commutativity) this limit involves tensionless open strings. Therefore it would be natural that in a dual open D-brane picture, this limit would become a zero-slope limit for open D-branes. In section 5 we showed that at long wave-length the resulting Dq-GT's are weakly coupled indicating a formulation in terms of some generalized gauge theory. For $q = 2, 3$ these theories have two length scales; one, set by $\theta_{\mathrm{D}q\text{-GT}}$, describing the non-commutative loop-space structure of the theory; and a second much smaller one, set by $g^2_{\mathrm{D}q\text{-GT}}$, determining the effective coupling of the theory. This suggests a field theoretical realization with finite (loop-space) non-commutativity while being in a region of weak effective coupling at fixed energy. On the other hand, for $q = 1$ the D1-GT may be considered at any length scale, so it should be possible to remove the non-commutativity and go to an undeformed theory in four dimensions, which should be some extension of ordinary gauge theory.

We have also proposed S-duals of the D3-GT and six-dimensional NCYM theory in the form of D3-$\widetilde{\mathrm{GT}}$ and D1-$\widetilde{\mathrm{GT}}$ on the type-IIB NS5-brane, and their T-duals D0-$\widetilde{\mathrm{GT}}$, D2-$\widetilde{\mathrm{GT}}$ and D4-$\widetilde{\mathrm{GT}}$ on the type-IIA NS5-brane. These theories are tensionless limits of non-commutative little string theory, which are strongly coupled at long wave-length. However, the spatial non-commutativity length scale can be taken to be much smaller, leaving room for a phase where we expect that these theories have weakly coupled formulations in terms of generalized gauge theories with spatial (loop-space) non-commutativity on the IIB NS5-brane and tensor analogs on the IIA NS5-brane. In the limit of vanishing non-commutativity we expect these theories to be related to the tensionless little string theories [20, 36].

We also expect that the open D-brane theories could avoid some of the non-perturbative issues that one would associate with quantizing critical open M-theory membranes or finding other field theoretical constructions of OM theory such as the generalized tensor theories discussed above. Nevertheless we found that the interactions of open D-brane based theories on the IIA NS5- and D4-branes seem close to the OM theory interactions, in the sense that the effective OM coupling can be kept fixed at fixed energy in the limit of small radius where the open D-brane theories become perturbative. Whether this brings good news for attempts at understanding the OM theory, or bad news for understanding open D2-branes, and other open Dq-branes remains to be seen.

Some of the results in this paper — in particular on the $\widetilde{\mathrm{OD}q}$ theories in section 4 — overlaps with [37] which appeared while we were preparing the manuscript.

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A. Conventions for S-duality, T-duality and lift to 11-dimensions

In this appendix we give the conventions we have used for S-duality, T-duality and lift of type-IIA solutions to 11-dimensions.

The S-duality rules for a type-IIB solution (in the string frame) is given by (the axion is set to zero):

$$\begin{aligned} d\tilde{s}^2 &= e^{-\phi} ds^2\,, \qquad e^{\tilde{\phi}} = e^{-\phi}\,, \\ \tilde{B} &= C_2\,, \qquad \tilde{C}_2 = -B\,, \\ \tilde{C}_4 &= C_4 + B \wedge C_2\,. \end{aligned} \tag{A.1}$$

For uplifting of a type-IIA solution to eleven dimensions we use the following relations:

$$\begin{aligned} \frac{ds_{11}^2}{\ell_p^2} &= e^{-2\phi/3}\frac{ds_{\mathrm{IIA}}^2}{\alpha'} + e^{4\phi/3}\left(\frac{dx^{11}}{R} - \frac{C_1}{\sqrt{\alpha'}}\right)^2, \\ \frac{A_3}{\ell_p^3} &= \frac{C_3}{(\alpha')^{3/2}} + \frac{dx^{11}}{R} \wedge \frac{B_2}{\alpha'}\,, \end{aligned} \tag{A.2}$$

where x^{11} has radius R and ℓ_p is the eleven-dimensional Planck length, which are given by $R = g\sqrt{\alpha'}$ and $\ell_p = g^{1/3}\sqrt{\alpha'}$, where g is the asymptotic value of the dilaton.

For T-duality between IIA and IIB solutions we use the Buscher rules [38] (in the string frame)

$$\begin{aligned} \tilde{g}_{yy} &= \frac{1}{g_{yy}}\,, \qquad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{g_{\mu y}g_{\nu y} - B_{\mu y}B_{\nu y}}{g_{yy}}\,, \\ \tilde{B}_{\mu y} &= \frac{g_{\mu y}}{g_{yy}}\,, \qquad \tilde{B}_{\mu\nu} = B_{\mu\nu} - \frac{B_{\mu y}g_{\nu y} - g_{\mu y}B_{\nu y}}{g_{yy}}\,, \\ \tilde{g}_{\mu y} &= \frac{B_{\mu y}}{g_{yy}}\,, \qquad e^{2\tilde{\phi}} = \frac{e^{2\phi}}{g_{yy}}\,, \end{aligned} \tag{A.3}$$

where y denotes the Killing coordinate with respect to which the T-dualization is applied, while μ,ν denote any coordinate direction other then y. The RR p-form fields transforms as [39]:

$$\begin{aligned} \tilde{C}_{(p)\mu\cdots\nu\rho y} &= C_{(p-1)\mu\cdots\nu\rho} - (p-1)\frac{C_{(p-1)[\mu\cdots\nu|y|}g_{\rho]y}}{g_{yy}}\,, \\ \tilde{C}_{(p)\mu\cdots\nu\rho\sigma} &= C_{(p+1)\mu\cdots\nu\rho\sigma y} + pC_{(p-1)[\mu\cdots\nu\rho}B_{\sigma]y} + \\ &\quad + p(p-1)\frac{C_{(p-1)[\mu\cdots\nu|y|}B_{\rho|y|}g_{\sigma]y}}{g_{yy}}\,. \end{aligned} \tag{A.4}$$

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JHEP06(2001)008